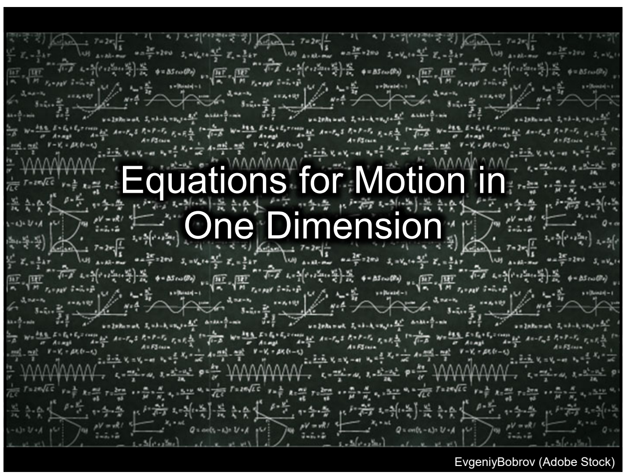
One Dimensional Motion



- The motion of an object can be described mathematically by using equations showing the displacement, velocity, and acceleration of an object at a given time.
- We will derive these equations for motion in one dimension with the assumption that acceleration is constant.
  - $a_{avg} = a = constant$

- Notation and assumptions:
  - $t_0 = 0$, so t will represent the final time.
  - We will use the x direction for convenience.
    - Position and velocity will be represented as follows:
      - x_0, x – initial and final position
      - v_0, v – initial and final velocity

- First, we rearrange the equation defining acceleration.

$$a_x = \frac{\Delta v_x}{\Delta t}$$

$$a_x = \frac{v_x - v_{x0}}{t}$$

$$\boxed{v_x = v_{x0} + a_x t} \quad (1)$$

- Now, rearrange the equation defining velocity.

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$$

For constant acceleration

$$v_x = \frac{v_{x0} + v_x}{2}$$

Set the equations equal to each other.

$$\frac{x - x_0}{t} = \frac{v_{x0} + v_x}{2} \quad (2)$$

- Substitute equation (1) into (2) and solve for x.

$$\frac{x - x_0}{t} = \frac{v_{x0} + v_{x0} + a_x t}{2}$$

$$x - x_0 = v_{x0} t + \frac{1}{2} a_x t^2$$

$$\boxed{x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2}$$

- Solve the equation (1) for time and then substitute it into equation (2).

$$t = \frac{v_x - v_{x0}}{a_x} \qquad \frac{x - x_0}{t} = \frac{v_{x0} + v_x}{2}$$

$$\frac{(x - x_0)}{(v_x - v_{x0})} a_x = \frac{v_{x0} + v_x}{2}$$

- Simplify and solve for v_x^2.

$$(v_x + v_{x0})(v_x - v_{x0}) = 2a_x(x - x_0)$$

$$v_x^2 - v_{x0}^2 = 2a_x(x - x_0)$$

$$\boxed{v_x^2 = v_{x0}^2 + 2a_x(x - x_0)}$$

Example 1

A rolling ball starts with a speed of 2.0 m/s and slows at a constant rate of 0.50 m/s^2. Calculate its velocity after 2.0 s.

$$v_x = v_{x0} + a_x t$$

$$v_x = 2 + (-0.5)2$$

$$\boxed{v_x = 1.0 \text{ m/s}}$$

Example 2

A car accelerates at a constant rate from 15 m/s to 25 m/s while it travels 125 m. How long does it take to achieve this speed?

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$a_x = \frac{v_x^2 - v_{x0}^2}{2(x - x_0)} = \frac{25^2 - 15^2}{2(125)}$$

$$a_x = 1.6\ \mathrm{m/s}^2$$

$$v_x = v_{x0} + a_x t$$

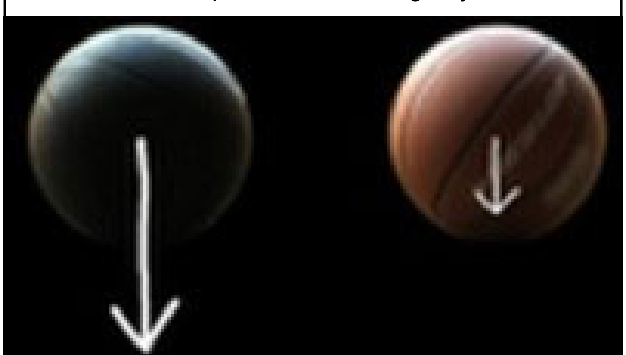
$$t = \frac{v_x - v_{x0}}{a_x} = \frac{25 - 15}{1.6}$$

$$\boxed{t = 6.3\ \mathrm{s}}$$

Vertical Motion

OpenClipart - Vectors

Misconceptions About Falling Objects

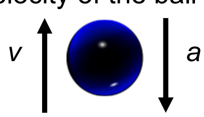


https://youtu.be/_mCC-68LyZM

- When air resistance is not a factor, **all** objects near Earth's surface fall with an acceleration of about 9.8 m/s^2.
  - Although this value decreases slightly with increasing altitude, it may be assumed to be essentially constant.
- The value of 9.8 m/s^2 is labeled **g** and is referred to as the **acceleration due to gravity**.
- Since gravity pulls objects towards the earth's surface, this acceleration is **always** down (negative).

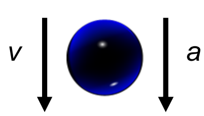
What happens when a ball is thrown straight up in the air?

- On its way up, the ball slows down
  - The acceleration due to gravity is in the opposite direction of the velocity of the ball



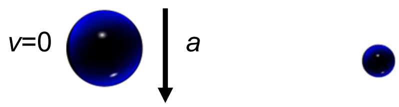
Credit: Sabrog (public domain)

- On its way down, the ball speeds up
  - The acceleration due to gravity is in the same direction as the velocity of the ball



Credit: Sabrog (public domain)

- What happens to the ball at the very top of its path?
  - It stops
- What is the acceleration at that point?
  - It is still the acceleration due to gravity and it is still down
  - The **direction** of the ball is changing instead of the speed.



Credit: Sabrog (public domain)

Brian Cox visits the world's biggest vacuum (Human Universe – BBC)

https://youtu.be/E43-CfukEgs

Apollo 15 Proves Galileo Correct

https://youtu.be/ZVfhztmK9zI

Example 1

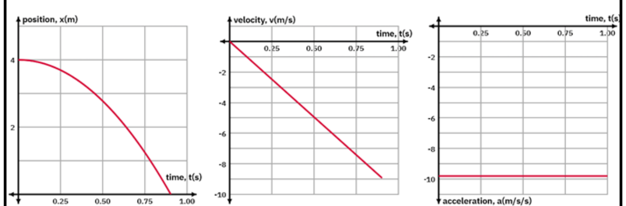
A ball is dropped from a height of 4.0 m. What is its velocity just before it hits the ground?

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$v_x = \sqrt{2a_x x} = \sqrt{2(-9.8)(-4.0)}$

$v_x = -8.9 \text{ m/s}$ or $v_x = 8.9 \text{ m/s down}$

- Graphing the motion.



Example 2

A ball is thrown straight up in the air with a velocity of 5.0 m/s from a height of 2.0 m.

a) How high above the ground does the ball go?

b) How long is the ball in the air?

a) How high above the ground does the ball go?

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$x = \frac{-v_{x0}^2}{2a_x} + x_0 = \frac{-(5^2)}{2(-9.8)} + 2$

$\boxed{x = 3.3\text{ m}}$

b) How long is the ball in the air?

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

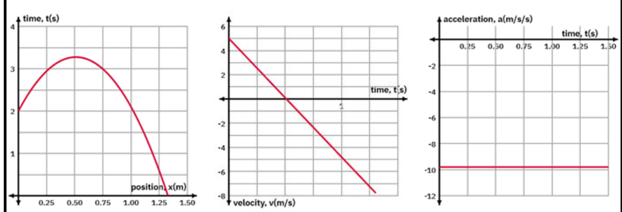
$0 = 2 + 5t + \frac{1}{2}(-9.8)t^2$

$4.9t^2 - 5t - 2 = 0$

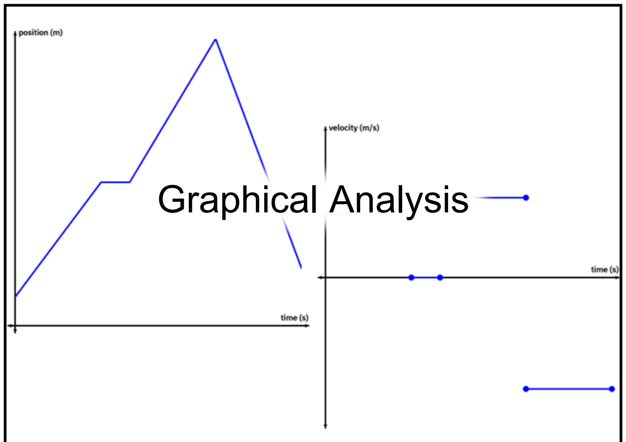
$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4.9)(-2)}}{2(4.9)} = 1.3, \cancel{-0.31}$

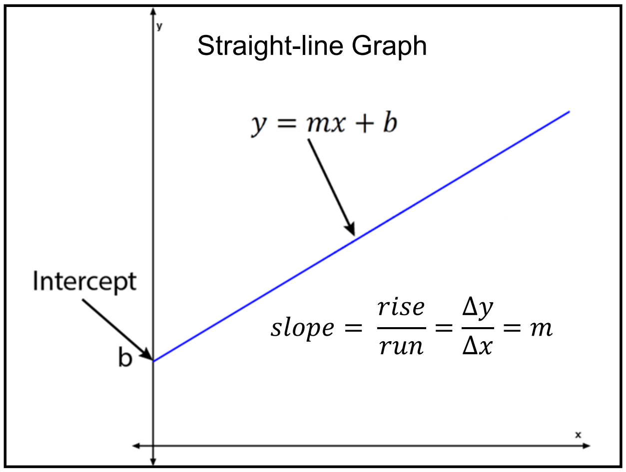
$\boxed{t = 1.3\text{ s}}$

- Graphing the motion.

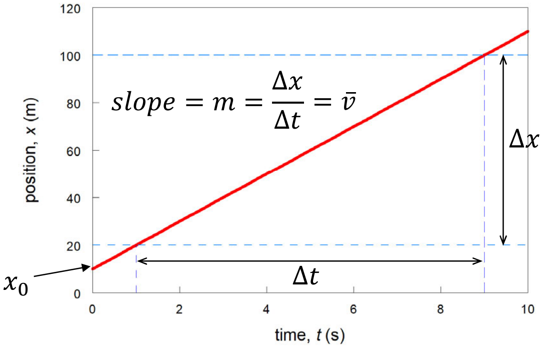


Graphical Analysis





The position-time graph of an object with constant velocity ($a = 0$).



- Substituting into the equation of a line gives

$$y = mx + b$$
$$x = \bar{v}t + x_0$$

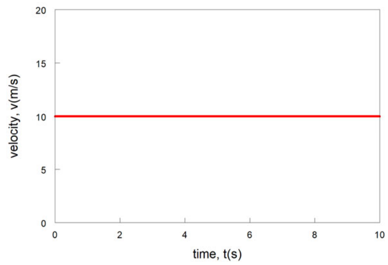
- Since velocity is constant $\bar{v} = v_{x0}$

$$x = x_0 + v_{x0}t$$

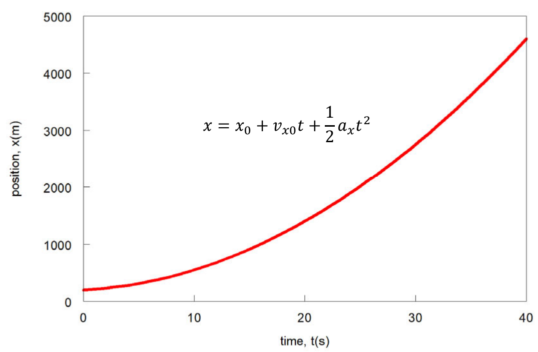
Note: This is the equation that we derived earlier with $a = 0$.

- If the acceleration is zero, then a velocity-time graph is a constant line.
- In this example,

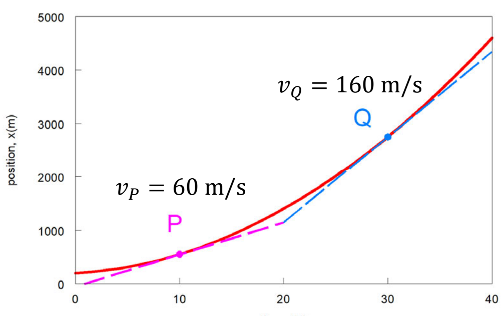
$$\frac{\Delta x}{\Delta t} = \frac{100 - 20}{9 - 1} = 10 \text{ m/s}$$



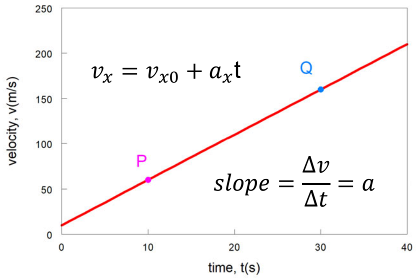
The position-time graph of an object that has a constant acceleration not equal to zero.



- The velocity at any point (instantaneous velocity) is the slope of the tangent line at that point.



- Calculating the velocity at each point and plotting it against time gives us a velocity-time graph.



- The slope of the graph is the acceleration.

Relative Motion

- Suppose a person is sitting in a train moving east at 10 m/s.
- If we choose east as the positive direction and Earth as the reference frame, then the train has a velocity of 10 m/s east relative to the earth (v_{TE}).

- Now say the person walks towards the back of the train (east) at 2 m/s (v_{PT}).
- This is the velocity relative to the reference frame of the train.
- To calculate the passenger's velocity relative to the earth (v_{PE}), we add the two vectors.

$$v_{PE} = v_{TE} + V_{PT}$$
$$v_{PE} = 10 - 2$$
$$v_{PE} = 8 \text{ m/s east}$$